Technical Notes

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Radiation View Factors from Differential Plane Sources to Disks—A General Formulation

M. H. N. Naraghi*
Manhattan College, Riverdale, New York

Introduction

THE objective of this work is to present a general formulation for evaluating diffuse radiation view factors from planar point sources to disks. The contour integral method[1] is used to derive two equations for the view factor from differential plane source dA_1 to disk A_2. Two exact solutions are obtained here; one corresponds to the case when the plane of differential area dA_1 does not intersect the plane of disk A_2 and the other one corresponds to when the plane of differential area dA_1 intersects disk A_2. There are a number of existing solutions for view factors from planar point sources to disks.[2-6] The present formulation is the most general, such that all previous solutions are considered limiting cases. It covers all possible positions of differential planar point sources relative to disks.

Analysis

Consider the configuration shown in Fig. 1. The differential area dA_1 is located at the origin of the coordinate system and is perpendicular to the yz plane. The normal unit vector n_1 is therefore in the yz plane, making an angle θ with the z axis. The angle ϕ is related to the coordinates of the disk center in the xy plane, i.e., a and b. For any possible relative position of the differential area dA_1 with respect to the disk A_2, the respective values of a, b, h, and angle θ can be determined.

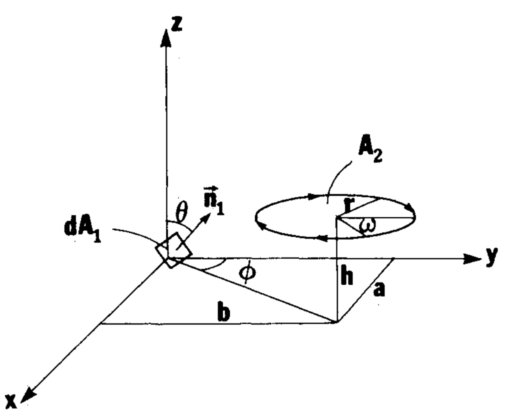

Fig. 1 Planar point source and disk configuration.

The view factor from differential area dA_1 to disk A_2 can be evaluated based on the contour integral method given by[1]

$$F_{dA_1-A_2} = \ell_1 \oint_C \frac{(z_2-z_1)dy_2-(y_2-y_1)dz_2}{2\pi L^2} + m_1 \oint_C \frac{(x_2-x_1)dz_2-(z_2-z_1)dx_2}{2\pi L^2} + n_1 \oint_C \frac{(y_2-y_1)dx_2-(x_2-x_1)dy_2}{2\pi L^2} \tag{1}$$

where

$$\ell_1 = 0, \quad m_1 = \sin\theta, \quad n_1 = \cos\theta$$

$$x_1 = y_1 = z_1 = 0$$

$$x_2 = a + r\sin\omega \qquad dx_2 = r\cos\omega d\omega$$

$$y_2 = b + r\cos\omega \qquad dy_2 = -r\sin\omega d\omega$$

$$z_2 = h \qquad dz_2 = 0$$

and

$$L^2 = x_2^2 + y_2^2 + z_2^2 = a^2 + b^2 + h^2 + r^2 + 2r(a\sin\omega + b\cos\omega)$$

Integrating Eq. (1) yields

$$F_{dA_1-A_2} = \frac{hb}{2(a^2+b^2)} \times \left[\frac{a^2+b^2+h^2+r^2}{\sqrt{(a^2+b^2+h^2+r^2)^2-4r^2a^2-4r^2b^2}} - 1\right]\sin\theta + \frac{1}{2}\left[1+\frac{r^2-a^2-b^2-h^2}{\sqrt{(a^2+b^2+h^2+r^2)^2-4r^2a^2-4r^2b^2}}\right]\cos\theta \tag{2}$$

The above equation corresponds to the case when the disk is fully visible from differential area dA_1. For certain values of angle θ, a part of the disk is visible from dA_1. In this case, the contour of the disk consists of two parts, a straight line and a partial circle (see Fig. 2). For the circle, the procedure is the same as in the previous case, except that the limits of integration are

$$\omega_1 = -\pi + \cos^{-1}\left(\frac{h\cot\theta + b}{r}\right)$$

and

$$\omega_2 = \pi - \cos^{-1}\left(\frac{h\cot\theta + b}{r}\right)$$

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*Assistant Professor, Department of Mechanical Engineering. Member AIAA.

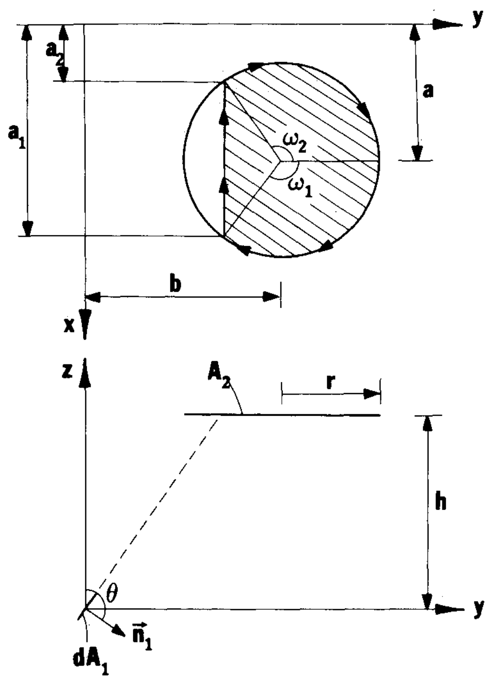

Fig. 2 Partial disk visible from planar point source.

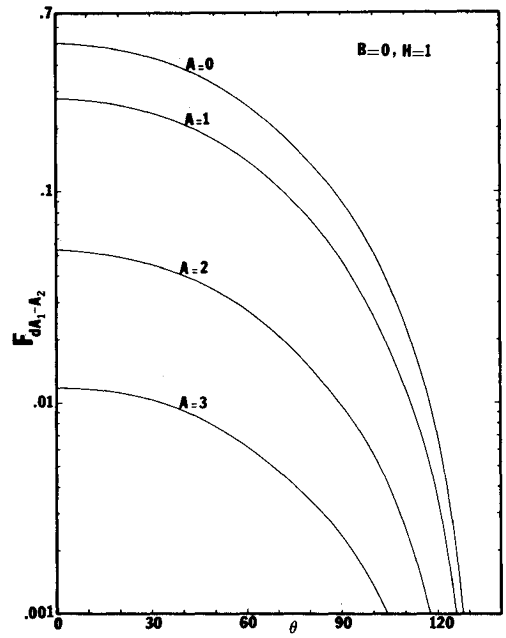

Fig. 3 View factors from a planar point source to a disk vs angle θ when $B=0$ and $H=1$ and A is a parameter.

For the line

$$x_2 = x \qquad dx_2 = dx$$

$$y_2 = -h\cot\theta \qquad dy_2 = 0$$

$$z_2 = h \qquad dz_2 = 0$$

$$L^2 = x_2^2 + y_2^2 + z_2^2 = x^2 + h^2\csc^2\theta$$

and the limits of integration are given by

$$a_1 = a + \sqrt{r^2 - (h\cot\theta + b)^2} \text{ and } a_2 = a - \sqrt{r^2 - (h\cot\theta + b)^2}$$

Substituting the above quantities into Eq. (1) after integration over the contour shown in Fig. 2, we obtain

$$F_{dA_1-A_2} = -\frac{ha\sin\theta}{4\pi(a^2+b^2)} + \ell n\left[\frac{a^2+b^2+h^2+r^2+2(a\sqrt{r^2-(h\cot\theta+b)^2}-bh\cot\theta-b^2}{a^2+b^2+h^2+r^2+2(-a\sqrt{r^2-(h\cot\theta+b)^2}-bh\cot\theta-b^2}\right]$$

$$+\frac{1}{2\pi}\left(\cos\theta - \frac{hb\sin\theta}{a^2+b^2}\right)\left[\pi - \cos^{-1}\left(\frac{h\cot\theta+b}{r}\right)\right] + \frac{[hb(a^2+b^2+h^2+r^2)/(a^2+b^2)]\sin\theta + (r^2-a^2-b^2-h^2)\cos\theta}{2\pi\sqrt{(a^2+b^2+h^2+r^2)^2-4r^2a^2-4r^2b^2}}$$

$$\times\left\{\tan^{-1}\left[\frac{2ra+(a^2+b^2+h^2+r^2-2rb)\cot(\cos^{-1}[(h\cot\theta+b)/r]/2)}{\sqrt{(a^2+b^2+h^2+r^2)^2-4r^2a^2-4r^2b^2}}\right]\right.$$

$$\left.-\tan^{-1}\left[\frac{2ra-(a^2+b^2+h^2+r^2-2rb)\cot(\cos^{-1}[(h\cot\theta+b)/r]/2)}{\sqrt{(a^2+b^2+h^2+r^2)^2-4r^2a^2-4r^2b^2}}\right]\right.$$

$$+\frac{1}{2\pi}\left[\tan^{-1}\left(\frac{[a+\sqrt{r^2-(h\cot\theta+b)^2}]\sin\theta}{h}\right) - \tan^{-1}\left(\frac{[a-\sqrt{r^2-(h\cot\theta+b)^2}]\sin\theta}{h}\right)\right] \tag{3}$$

This equation is valid when $r^2 - (h \cot\theta + b)^2 > 0$, i.e., when the plane of dA_1 intersects the disk. Using dimensionless variables $A = a/r$, $B = b/r$, and $H = h/r$, Eqs. (2) and (3) become

$$F_{dA_1-A_2} = \frac{HB}{2(A^2+B^2)} \times \left[\frac{A^2+B^2+H^2+1}{\sqrt{(A^2+B^2+H^2+1)^2-4A^2-4B^2}}\right]\sin\theta \; \frac{1}{2}\left[1+\frac{1-A^2-B^2-H^2}{\sqrt{(A^2+B^2+H^2+1)^2-4A^2-4B^2}}\right]\cos\theta \tag{4}$$

When the disk is fully visible and

$$F_{dA_1-A_2} = -\frac{HA}{4\pi(A^2+B^2)} + \ell n\left[\frac{A^2+B^2+H^2+r^2+2(A\sqrt{1-(H\cot\theta+B)^2}-BH\cot\theta-B^2}{A^2+B^2+H^2+1+2(-A\sqrt{1-(H\cot\theta+B)^2}-BH\cot\theta-B^2}\right]$$

$$+\frac{1}{2\pi}\left(\cos\theta-\frac{HB\sin\theta}{A^2+B^2}\right)[\pi-\cos^{-1}(H\cot\theta+B)]+\frac{[HB(A^2+B^2+H^2+1)/(A^2+B^2)]\sin\theta+(1-A^2-B^2-H^2)\cos\theta}{2\pi\sqrt{(A^2+B^2+H^2+1)^2-4A^2-4B^2}}$$

$$\times\left\{\tan^{-1}\left[\frac{2A+(A^2+B^2+H^2+1-2B)\cot\{[\cos^{-1}(H\cot\theta+B)]/2\}}{\sqrt{A^2+B^2+H^2+1)^2-4A^2+4B^2}}\right]\right.$$

$$\left.-\tan^{-1}\left[\frac{2A-(A^2+B^2+H^2+1-2B)\cot\{[\cos^{-1}(H\cot\theta+B)]/2\}}{\sqrt{(A^2+B^2+H^2+1)^2-4A^2-4B^2}}\right]\right\}$$

$$+\frac{1}{2\pi}\left[\tan^{-1}\{[A+\sqrt{1-(H\cot\theta+B)^2}]\sin\theta\}-\tan^{-1}\{[A-\sqrt{1-(H\cot\theta+B)^2}]\sin\theta]\}\right. \tag{5}$$

when the disk is partially visible from dA_1, i.e., $l-(H\cot\theta+B)^2>0$.

Figure 3 shows numerical values of view factors based on the above formulation vs cone angle θ when $H=1$ and $B=0$ and A is a parameter. Figure 4 is similar to Fig. 3, except that $B=1$ in Fig. 4. As is shown in these figures, the view factors are at a maximum point at certain angle θ. In Fig. 3, Eq. (4) is used to evaluate view factors when $\theta<45$ deg and Eq. (5) when $\theta>45$ deg. Similarly, in Fig. 4, Eq. (4) is used when $\theta<90$ deg and Eq. (5) when $\theta>90$ deg.

When $A=0$, Eqs. (4) and (5) reduce to

$$F_{dA_1-A_2} = \frac{H}{2B}\left[\frac{B^2+H^2+1}{\sqrt{(B^2+H^2+1)^2-4B^2}}-1\right]\sin\theta + \frac{1}{2}\left[1+\frac{l-B^2-H^2}{\sqrt{(B^2+H^2+1)^2-4B^2}}\right]\cos\theta \tag{6}$$

when the disk is fully visible from dA_1 and

$$F_{dA_1-A_2} = \frac{1}{2\pi}\left(\cos\theta-\frac{H}{B}\sin\theta\right)(\pi-\cos^{-1}[H\cot\theta+B]) + \frac{H(B^2+H^2+1)/B\sin\theta+(l-B^2-H^2)\cos\theta}{\pi\sqrt{(B^2+H^2+1)^2-4B^2}}$$

$$\times\tan^{-1}\left\{\sqrt{\frac{B^2+H^2+1-2B}{B^2+H^2+1+2B}}\cot\left[\frac{\cos^{-1}(H\cot\theta+B)}{2}\right]\right\} + \frac{1}{\pi}\tan^{-1}[\sqrt{1-(H\cot\theta+B)^2}\sin\theta] \tag{7}$$

when the disk is partially visible from dA_1 or $l-(H\cot\theta+B)^2>0$. Formulations given by Eqs. (6) and (7) are identical to those given by Naraghi and Chung.[4] Results of Juul[3] can be obtained from the present formulation by setting $B=0$ in Eqs. (6) and (7), which are given by

$$F_{dA_1-A_2} = \frac{1}{1+H^2}\cos\theta \tag{8}$$

$$F_{dA_1-A_2} = \frac{1}{\pi}\left\{-\frac{H\sqrt{l-H^2\cot^2\theta}}{1+H^2}\sin\theta \pm\frac{1}{1+H^2}[1-\cos^{-1}(H\cot\theta)]\cos\theta +\frac{1}{\pi}\tan^{-1}[\sqrt{1-H^2\cot^2\theta}\sin\theta]\right\} \tag{9}$$

when $1-H^2\cot^2\theta>0$

Formulations given by Eqs. (2) and (3) can be used to determine the view factor between a disk and a finite surface. The

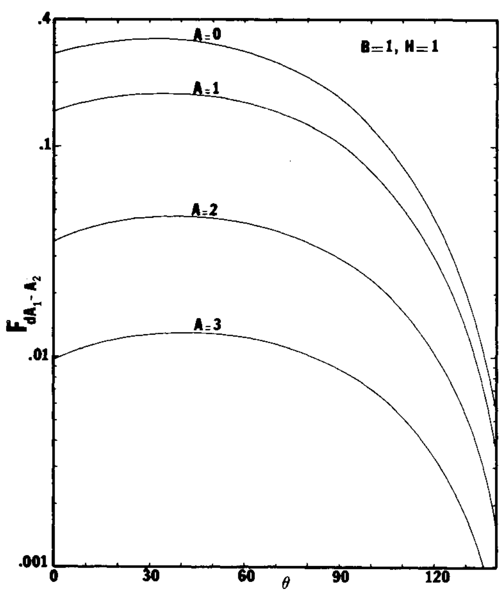

Fig. 4 View factors from a planar point source to a disk vs angle θ when $B=1$ and $H=1$ and A is a parameter.

view factor from the disk to differential area dA_1 is given by

$$dF_{A_2-dA_1}=\frac{dA_1}{\pi R^2}F_{dA_1-A_2}$$

Then, the view factor from the disk to a finite surface can be calculated from

$$F_{A_2-A_1}=\int_{A_1}dF_{A_2-dA_1}=\int_{A_1}F_{dA_1-A_2}\frac{dA_1}{\pi R^2}$$

Conclusion

In conclusion, a general formulation for diffuse configuration factors from planar point sources to disks is presented here. This formulation can be used to evaluate view factors for all possible positions of differential areas relative to disks. The present formulation can be reduced to the existing formulation by assigning numerical values to a number of its parameters. Finally, the resulting equations can be used to evaluate view factors from disks to finite surfaces.

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Cavitation of Liquid Streams in a Vacuum

Melissa Orme* and E. P. Muntz†
University of Southern California, Los Angeles, California
and
H. Legge‡ and G. Koppenwallner§
DFVLR, Institute for Experimental Fluid Mechanics, Göttingen, Federal Republic of Germany

Introduction

As our activities in space increase, there is a growing requirement for accurate predictions of the results of injecting finite vapor pressure liquids into the space environment. Liquids may be exposed to space because of either accidents or of planned dumps. During experimental studies at the DFVLR Institute for Rarefied Gases at Göttingen,[1] the following behavior of a water jet in a vacuum was observed. The water jet that was formed in a tapered nozzle leaves it as a collimated jet and abruptly bursts into ice particles and water droplets downstream, forming a conical volume filled with stream fragments. Both the distance at which the bursting takes place and the angle defined by the cone of stream fragments depend on flow parameters such as the initial water temperature, the nozzle diameter, and the stagnation pressure. In this paper, comparisons are made between the authors' interpretations of the data obtained from the DFVLR and predictions of the fragment cone's angle and the bursting point's location for water jets injected into a vacuum.

Analysis

Phenomena that occur when a volatile liquid stream is injected into a vacuum have been studied by Fuchs and Legge[1] and Muntz and Orme.[2] For the purposes of this paper, we assume an axisymmetric cylindrical water jet emanating from a contoured nozzle as shown. The stream has a speed V_0, which is associated with the stagnation pressure p_0. As the fluid enters the vacuum, the pressure of the surface of the jet drops to about half of the vapor pressure of the fluid associated with the temperature of the jet. An increase in the evaporation rate that is a result of the rapid pressure drop causes the jet to cool at the surface while moving downstream from the nozzle exit. Despite surface cooling (which leads to a lowered vapor pressure) and the retarding effect of internal pressure generated by surface tension, a vapor bubble may expand in the stream under certain conditions of initial bubble size, stream diameter, and initial stream temperature. As a bubble continues to grow, it eventually reaches a critical size that causes the jet to burst into water droplets and ice fragments. The fragments form a conical envelope where the apex is defined as the burst point.

The equation of motion for the radius R_B of a bubble in a viscous liquid as a function of time is (see Ref. 3):

$$R_B\ddot{R}_B+\frac{3}{2}\dot{R}_B^2=\frac{\Delta p}{\rho_\ell}-\frac{2\sigma}{\rho_\ell R_B}-\frac{4\mu\dot{R}_B}{\rho_\ell R_B} \tag{1}$$

where

$$\Delta p=p_{vB}-\frac{\sigma}{a_0}-\frac{p_v}{2} \tag{2}$$

is the pressure difference driving the bubble expansion. This particular form of Δp is for a cylindrical stream and bubbles with vapor pressure p_{vB}. Here, σ is the surface tension, μ the coefficient of viscosity, ρ_ℓ the density of the liquid, and a_0 the stream radius. The vapor pressure p_v is a function of the temperature and is given by the Clausius–Clapeyron equation.

The reduction of surface temperature with time due to vaporization of both the external stream and the interior bubble wall were calculated by using an approximation that we developed for the integral relation for surface temperature due to Shultz and Jones as given by Fuchs and Legge.[1] The approximation is decribed in Ref. 2.

Equation (1) can be solved straightforwardly with the boundary condition that $\hat{R}_B=0$ at $\hat{t}=0$. Typical results of $\hat{R}_B$ plotted against $\hat{t}$ are illustrated in Ref. 2. Here, $\hat{R}_B$ is the nondimensional bubble radius (R_B/a_0) and $\hat{t}$ is the nondimensional time equal to tV_0/a_0, where V_0 is the stream speed and a_0 the stream radius.

Results

Bursting Cone Angles

To calculate the characteristic "cone angles" produced when a stream bursts, we have assumed that the bursting takes

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*Research Assistant. Student Member AIAA.
†Professor. Fellow AIAA.
‡Principal Scientist.
§Director, Rarefied Gas Dynamics.